Supplementary Material to "Firm Size and Diversification: Multiproduct Firms in Asymmetric Oligopoly"

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Abstract

This supplement provides details of extensive derivations in the linear-demand model. Part 1 refers to the Cournot case and part 2 to the Bertrand case.

In part 1, first, equations (A.8)-(A.11) in appendix (proof of Corollary 1) are derived. Second, Lemma A.1 (used for the proof of Proposition 2) is proven. Finally, basic properties of the functions D_i (total sales in stage 2 equilibrium) and M_i (mark-up in stage 2 equilibrium), which are used for the discussion of Corollary 1, are derived.

In part 2, first, equilibrium profits at stage 2 and mark-up, Π_i and M_i, respectively, are derived for the Bertrand-case. Then properties of these functions as stated in section 4.2 are derived by providing both analytical and numerical results. Finally, the positive relationship between product range and sales in the Bertrand case is established.

1 Cournot case under linear demand

Derivation of (A.8): From (A.7),

$$\Pi_i = \frac{N_i(\beta - \gamma + \gamma N_i)\Lambda_i^2}{Z_i^2}, \tag{B.1}$$

where

$$Z_i \equiv 2(\beta - \gamma + \gamma N_i) + [2(\beta - \gamma) + \gamma N_i]\Phi_{-i}. \tag{B.2}$$

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Note that

$$\Lambda_i = (1 + \Phi_{-i})\alpha_i - \sum_{j \neq i} \alpha_j \Gamma_j \tag{B.3}$$

and

$$\Phi_{-i} = \sum_{h \neq i} \Gamma_h = \sum_{h \neq i} \frac{\gamma N_h}{2(\beta - \gamma) + \gamma N_h} \tag{B.4}$$

are independent of N_i. Also note that $\partial Z_i / \partial N_i = \gamma(2 + \Phi_{-i})$. Thus,

$$
\begin{aligned}
\frac{\partial \Pi_i}{\partial N_i} &= \frac{\Lambda_i^2}{Z_i^4} \left\{ (\beta - \gamma + 2\gamma N_i) Z_i^2 - N_i(\beta - \gamma + \gamma N_i) 2 Z_i \frac{\partial Z_i}{\partial N_i} \right\} \\
&= \frac{\Lambda_i^2}{Z_i^3} \{ (\beta - \gamma + 2\gamma N_i) \left[2(\beta - \gamma + \gamma N_i) + (2(\beta - \gamma) + \gamma N_i) \Phi_{-i} \right] - \\
&\quad 2\gamma N_i (\beta - \gamma + \gamma N_i)(2 + \Phi_{-i}) \} \\
&= \frac{\Lambda_i^2}{Z_i^3} \{ 2(\beta - \gamma + 2\gamma N_i)(\beta - \gamma + \gamma N_i) - 4\gamma N_i(\beta - \gamma + \gamma N_i) + \\
&\quad [(\beta - \gamma + 2\gamma N_i)(2(\beta - \gamma) + \gamma N_i) - 2\gamma N_i(\beta - \gamma + \gamma N_i)] \Phi_{-i} \} \\
&= \frac{\Lambda_i^2(\beta - \gamma)}{Z_i^3} \{ 2(\beta - \gamma + \gamma N_i) + (2(\beta - \gamma) + 3\gamma N_i) \Phi_{-i} \} > 0. \tag{B.5}
\end{aligned}
$$

Substituting (B.2) into (B.5) confirms (A.8). ∎

Derivation of (A.9): Using (A.8), one obtains

$$
\begin{aligned}
\frac{\partial^2 \Pi_i}{\partial N_i^2} &= \frac{\Lambda_i^2(\beta - \gamma)}{Z_i^6} \left\{ (2\gamma + 3\gamma\Phi_{-i}) Z_i^3 - [2(\beta - \gamma + \gamma N_i) + (2(\beta - \gamma) + 3\gamma N_i) \Phi_{-i}] 3 Z_i^2 \frac{\partial Z_i}{\partial N_i} \right\} \\
&= \frac{\Lambda_i^2(\beta - \gamma)\gamma}{Z_i^4} \{ (2 + 3\Phi_{-i})(2(\beta - \gamma + \gamma N_i) + (2(\beta - \gamma) + \gamma N_i) \Phi_{-i}) - \\
&\quad 3 [2(\beta - \gamma + \gamma N_i) + [2(\beta - \gamma) + 3\gamma N_i] \Phi_{-i}(2 + \Phi_{-i})] \} \\
&= \frac{\Lambda_i^2(\beta - \gamma)\gamma}{Z_i^4} \{ -2(\beta - \gamma + \gamma N_i) + \Phi_{-i}[6(\beta - \gamma + \gamma N_i) + 2(2(\beta - \gamma) + \gamma N_i) - \\
&\quad 6(2(\beta - \gamma) + 3\gamma N_i)] + \Phi_{-i}^2[3(2(\beta - \gamma) + \gamma N_i) - 3(2(\beta - \gamma) + 3\gamma N_i)]\} \\
&= -\frac{2\Lambda_i^2(\beta - \gamma)\gamma}{Z_i^4} \{ \beta - \gamma + \gamma N_i + (\beta - \gamma + 5\gamma N_i)\Phi_{-i} + 3\gamma N_i \Phi_{-i}^2 \} < 0. \tag{B.6}
\end{aligned}
$$

Substituting (B.2) into (B.6) confirms (A.9). ∎

Derivation of (A.10): First, note that, for all $i, j \in \mathcal{I}, j \neq i$,

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$$\frac{\partial \Lambda_i}{\partial N_j} = \frac{2(\beta - \gamma)\gamma(\alpha_i - \alpha_j)}{[2(\beta - \gamma) + \gamma N_j]^2} \tag{B.7}$$

and

$$\frac{\partial \Phi_{-i}}{\partial N_j} = \frac{\partial \Gamma_j}{\partial N_j} = \frac{2(\beta - \gamma)\gamma}{[2(\beta - \gamma) + \gamma N_j]^2}, \tag{B.8}$$

according to (B.3) and (B.4), respectively; moreover, we have

$$\frac{\partial Z_i}{\partial N_j} = \frac{2(\beta - \gamma)\gamma[2(\beta - \gamma) + \gamma N_i]}{[2(\beta - \gamma) + \gamma N_j]^2}, \tag{B.9}$$

according to (B.2) and (B.8). Hence, using

$$\Pi_i = \frac{N_i(\beta - \gamma + \gamma N_i)\Lambda_i^2}{Z_i^2}, \tag{B.10}$$

one obtains, for $j \neq i$,

$$\begin{aligned}
\frac{\partial \Pi_i}{\partial N_j} &= \frac{N_i(\beta - \gamma + \gamma N_i)}{Z_i^4} \left\{ 2\Lambda_i \frac{\partial \Lambda_i}{\partial N_j} Z_i^2 - \Lambda_i^2 2 Z_i \frac{\partial Z_i}{\partial N_j} \right\} \\
&= \frac{4N_i(\beta - \gamma + \gamma N_i)\Lambda_i(\beta - \gamma)\gamma S_{i,j}}{[2(\beta - \gamma) + \gamma N_j]^2 Z_i^3},
\end{aligned} \tag{B.11}$$

where

$$\begin{aligned}
S_{i,j} &\equiv (\alpha_i - \alpha_j)\left[2(\beta - \gamma + \gamma N_i) + (2(\beta - \gamma) + \gamma N_i)\Phi_{-i}\right] - (2(\beta - \gamma) + \gamma N_i)\Lambda_i \\
&= 2(\alpha_i - \alpha_j)(\beta - \gamma + \gamma N_i) + [2(\beta - \gamma) + \gamma N_i][(\alpha_i - \alpha_j)\Phi_{-i} - \Lambda_i]. \tag{B.12}
\end{aligned}$$

Using (B.3), one finds

$$\begin{aligned}
S_{i,j} &= 2(\alpha_i - \alpha_j)(\beta - \gamma + \gamma N_i) - [2(\beta - \gamma) + \gamma N_i](\alpha_j \Phi_{-i} + \alpha_i - \sum_{j \neq i} \alpha_j \Gamma_j) \\
&= \gamma N_i(\alpha_i - \alpha_j) - [2(\beta - \gamma) + \gamma N_i]\left(\alpha_j(1 + \Phi_{-i}) - \sum_{j \neq i} \alpha_j \Gamma_j\right) \\
&= -[2(\beta - \gamma) + \gamma N_i]\left[(\alpha_j - \alpha_i)\Gamma_i + \alpha_j\left(1 + \Gamma_j + \sum_{h \neq i,j} \Gamma_h\right) - \alpha_j \Gamma_j - \sum_{h \neq i,j} \alpha_h \Gamma_h\right] \\
&= -[2(\beta - \gamma) + \gamma N_i]\left[\alpha_j\left(1 + \sum_{h \neq j} \Gamma_h\right) - \sum_{h \neq j} \alpha_h \Gamma_h\right] \\
&= -[2(\beta - \gamma) + \gamma N_i]\Lambda_j. \tag{B.13}
\end{aligned}$$

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Substituting (B.2) and (B.13) into (B.12) confirms (A.10). ∎

Derivation of (A.11): From (A.8), by making use of (B.7) and (B.9), one obtains, for $j \neq i$,

$$
\begin{aligned}
\frac{\partial^2 \Pi_i}{\partial N_i \partial N_j} &= \frac{\beta - \gamma}{Z_i^6} \{\{ (2(\beta - \gamma) + 3\gamma N_i)(\partial \Phi_{-i}/\partial N_j)\Lambda_i^2 + \\
&\quad [2(\beta - \gamma + \gamma N_i) + (2(\beta - \gamma) + 3\gamma N_i)\Phi_{-i}]2\Lambda_i \frac{\partial \Lambda_i}{\partial N_j}\} Z_i^3 - \\
&\quad \Lambda_i^2 [2(\beta - \gamma + \gamma N_i) + (2(\beta - \gamma) + 3\gamma N_i)\Phi_{-i}]3Z_i^2 \frac{\partial Z_i}{\partial N_j}\} \\
&= \frac{2(\beta - \gamma)^2 \gamma \Lambda_i}{Z_i^4 [2(\beta - \gamma) + \gamma N_j]^2} \{ 2(\alpha_i - \alpha_j)[2(\beta - \gamma + \gamma N_i) + (2(\beta - \gamma) + 3\gamma N_i)\Phi_{-i}]Z_i + \\
&\quad \Lambda_i Z_i (2(\beta - \gamma) + 3\gamma N_i) - \\
&\quad 3\Lambda_i [2(\beta - \gamma + \gamma N_i) + (2(\beta - \gamma) + 3\gamma N_i)\Phi_{-i}](2(\beta - \gamma) + \gamma N_i)\} \\
&= \frac{2(\beta - \gamma)^2 \gamma \Lambda_i}{Z_i^4 [2(\beta - \gamma) + \gamma N_j]^2} \times \\
&\quad \{ 2(\alpha_i - \alpha_j)[2(\beta - \gamma + \gamma N_i) + (2(\beta - \gamma) + 3\gamma N_i)\Phi_{-i}]Z_i + \Lambda_i T_{i,j}\}, \quad \text{(B.15)}
\end{aligned}
$$

where

$$
\begin{aligned}
T_{i,j} &\equiv [2(\beta - \gamma + \gamma N_i) + (2(\beta - \gamma) + \gamma N_i)\Phi_{-i}](2(\beta - \gamma) + 3\gamma N_i) - 3(2(\beta - \gamma) + \gamma N_i) \times \\
&\quad [2(\beta - \gamma + \gamma N_i) + (2(\beta - \gamma) + 3\gamma N_i)\Phi_{-i}] \\
&= -2[(2(\beta - \gamma) + \gamma N_i)(2(\beta - \gamma) + 3\gamma N_i)\Phi_{-i} + 4(\beta - \gamma + \gamma N_i)(\beta - \gamma)], \quad \text{(B.16)}
\end{aligned}
$$

i.e., $T_{i,j} < 0$. Substituting (B.16) into (B.15) yields (A.11). Hence, if $\alpha_i \leq \alpha_j$, then $\partial^2 \Pi_i / \partial N_i \partial N_j < 0$, $j \neq i$. ∎

Proof of Lemma A.1: First, note that $\alpha_i = \alpha$ implies $\Lambda_i = \alpha$ for all i. Thus, (A.8) and the definition of W imply that $\partial W / \partial \gamma < 0$ if

$$0 \quad > \quad -Z_i \left[2(\beta - \gamma + \gamma N_i) + (2(\beta - \gamma) + 3\gamma N_i)\,\Phi_{-i}\right] + (\beta - \gamma) \times$$

$$\left\{\left[2(N_i - 1) + (3N_i - 2)\Phi_{-i}\right]\left[2(\beta - \gamma + \gamma N_i) + (2(\beta - \gamma) + \gamma N_i)\Phi_{-i}\right] - \right.$$

$$\left[(2(\beta - \gamma) + \gamma N_i)(\partial\Phi_{-i}/\partial\gamma) + 2(N_i - 1) + \Phi_{-i}(N_i - 2)\right] \times$$

$$\left. 3\left[2(\beta - \gamma + \gamma N_i) + (2(\beta - \gamma) + 3\gamma N_i)\,\Phi_{-i}\right]\right\}, \tag{B.17}$$

with Z_i as defined in (B.2). Some tedios manipulations reveal that this condition can be rewritten as

$$0 \quad < \quad 3\,(\beta - \gamma)\,(\partial\Phi_{-i}/\partial\gamma)(2(\beta - \gamma) + \gamma N_i)\left[2(\beta - \gamma + \gamma N_i) + (2(\beta - \gamma) + 3\gamma N_i)\,\Phi_{-i}\right] +$$

$$4(\beta - \gamma + \gamma N_i)\left[4(\beta - \gamma + \gamma N_i) + 2\,(\beta - \gamma)\,(N_i - 1)\right] +$$

$$8\Phi_{-i}\left[(\beta - \gamma)\,(N_i - 1)(\beta - \gamma + 2\gamma N_i) + \gamma N_i(\beta - \gamma + \gamma N_i)\right] +$$

$$\Phi_{-i}^2\left[3\gamma N_i^2(2\beta - \gamma) - 4(\beta - \gamma)(\beta - \gamma + 2\gamma N_i)\right]. \tag{B.18}$$

Lemma A.1 holds if (B.18) is fulfilled in the case where $N_i = N^*$ for all i. Recalling that $\Phi_{-i} = \sum_{j \neq i} \Gamma_j$ and $\Gamma_j = \gamma N_j / \left[2(\beta - \gamma) + \gamma N_j\right]$, and setting $N_i = N^*$, one finds

$$\Phi_{-i} = \frac{\gamma N^*(I - 1)}{2(\beta - \gamma) + \gamma N^*} \text{ and } \frac{\partial\Phi_{-i}}{\partial\gamma} = \frac{2\beta N^*(I - 1)}{\left[2(\beta - \gamma) + \gamma N^*\right]^2}. \tag{B.19}$$

Note that the second and third line of (B.18) are positive. Thus, using (B.19) implies that a sufficient condition for (B.18) - when at $N_i = N^*$ for all i - is

$$0 \quad < \quad \frac{6\,(\beta - \gamma)\,\beta N^*(I - 1)}{2(\beta - \gamma) + \gamma N^*}\left[2(\beta - \gamma + \gamma N^*) + \frac{\gamma N^*(I - 1)\,(2(\beta - \gamma) + 3\gamma N^*)}{2(\beta - \gamma) + \gamma N^*}\right] +$$

$$\left(\frac{\gamma N^*(I - 1)}{2(\beta - \gamma) + \gamma N^*}\right)^2\left[3\gamma(N^*)^2(2\beta - \gamma) - 4(\beta - \gamma)(\beta - \gamma + 2\gamma N^*)\right]. \tag{B.20}$$

Straightforward algebra reveals that (B.20) is fulfilled, which confirms Lemma A.1. ∎

Properties of $D_i(\mathbf{N}, \boldsymbol{\alpha}, \cdot)$ and $M_i(\mathbf{N}, \boldsymbol{\alpha}, \cdot)$: In section 3, we decomposed profits of a firm i in stage 2 equilibrium, $\Pi_i(\mathbf{N}, \boldsymbol{\alpha}, \cdot)$, into the product between equilibrium demand, $D_i(\mathbf{N}, \boldsymbol{\alpha}, \cdot) = N_i X_i(\mathbf{N}, \boldsymbol{\alpha}, \cdot)$, and equilibrium mark-up, $M_i(\mathbf{N}, \boldsymbol{\alpha}, \cdot) = (\beta - \gamma + \gamma N_i) X_i(\mathbf{N}, \boldsymbol{\alpha}, \cdot)$. The following analysis formally derives the properties of these two functions, $D_i(\mathbf{N}, \boldsymbol{\alpha}, \cdot)$ and $M_i(\mathbf{N}, \boldsymbol{\alpha}, \cdot)$, which have been used in the discussion of Corollary 2. We obtain the following results.

Corollary B.1. (Properties of $D_i(\mathbf{N}, \boldsymbol{\alpha}, \cdot)$). *For all $i, j \in \mathcal{I}$, $j \neq i$, we have*

(i) $\partial D_i / \partial N_i > 0$ and $\partial^2 D_i / \partial N_i^2 < 0$,

(ii) $\partial D_i / \partial N_j < 0$,

(iii) $\partial D_i / \partial \alpha_i > 0$, $\partial D_i / \partial \alpha_j < 0$,

(iv) $\partial^2 D_i / \partial N_i \partial \alpha_i > 0$ and $\partial^2 D_i / \partial N_i \partial \alpha_j < 0$, and

(v) if $\alpha_i \leq \alpha_j$ or if $(\alpha_i - \alpha_j)$ sufficiently small, then $\partial^2 D_i / \partial N_i \partial N_j < 0$.

Proof. First, note from (A.6) that $\partial X_i / \partial N_i = -\lambda_{i,j} X_i / N_i$, and thus, $\partial D_i / \partial N_i = X_i + N_i \partial X_i / \partial N_i = (1 - \lambda_{i,j}) X_i$, where

$$\lambda_{i,j} \equiv \frac{\gamma N_i (2 + \Phi_{-i})}{2(\beta - \gamma)(1 + \Phi_{-i}) + \gamma N_i (2 + \Phi_{-i})}. \tag{B.21}$$

Since $\lambda_{i,j} \in (0, 1)$, we have $\partial D_i / \partial N_i > 0$. Moreover, $\partial^2 D_i / \partial N_i^2 = (1 - \lambda_{i,j}) \partial X_i / \partial N_i - X_i \partial \lambda_{i,j} / \partial N_i < 0$, since $\partial X_i / \partial N_i < 0$, $\lambda_{i,j} \in (0, 1)$ and $\partial \lambda_{i,j} / \partial N_i > 0$, according to (B.25). This proves part (i) of Corollary B.1. To prove part (ii), note that $\partial D_i / \partial N_j = N_i \partial X_i / \partial N_j$, $j \neq i$. Using (A.6), (B.7) and (B.8), it is straightforward to show that this implies

$$\frac{\partial D_i}{\partial N_j} = -\frac{2\gamma(\beta - \gamma) N_i S_{i,j}}{[2(\beta - \gamma) + \gamma N_j]^2 Z_i^2}, \tag{B.22}$$

$j \neq i$, where Z_i and $S_{i,j}$ are given by (B.2) and (B.12), respectively. According to (B.13), we have $S_{i,j} < 0$, thus, confirming $\partial D_i / \partial N_j < 0$, $j \neq i$. To prove part (iii) of Corollary B.1, first, note that $X_i = \Lambda_i / Z_i$, according to (A.6) and (B.2). Thus, part (iii) directly follows from $\partial \Lambda_i / \alpha_i > 0$ and $\partial \Lambda_i / \alpha_j < 0$, $j \neq i$, according to (B.3), and the fact that $\partial D_i / \partial \alpha_j = N_i \partial X_i / \partial \alpha_j$. Recalling $\partial D_i / \partial N_i = (1 - \lambda_{i,j}) X_i$, part (iv) follows by similar considerations, together with the fact that $\lambda_{i,j}$ is independent of α_i or α_j, respectively, according to (B.21). To prove part (v), first, note that $\partial D_i / \partial N_i = 2(\beta - \gamma) \Lambda_i (1 + \Phi_{-i}) / Z_i^2$,

according to (B.2), (B.21) and $X_i = \Lambda_i/Z_i$. Using this together with (B.7)-(B.9), one obtains, after some manipulations, that, for $j \neq i$,

$$\frac{\partial^2 D_i}{\partial N_i \partial N_j} = \frac{4\gamma(\beta - \gamma)^2 N_i \{(\alpha_i - \alpha_j)(1 + \Phi_{-i})Z_i - \Lambda_i [\Phi_{-i}(2(\beta - \gamma) + \gamma N_i) + 2(\beta - \gamma)]\}}{[2(\beta - \gamma) + \gamma N_j]^2 Z_i^3}. \tag{B.23}$$

Recalling that $\Lambda_i > 0$ in interior equilibrium confirms part (v). This concludes the proof of Corollary B.1. ∎

Corollary B.2. (Properties of $M_i(\mathbf{N}, \boldsymbol{\alpha}, \cdot)$). *For all $i, j \in \mathcal{I}$, $j \neq i$, we have*

(i) $\partial M_i / \partial N_i > 0$ and $\partial^2 M_i / \partial N_i^2 < 0$,

(ii) $\partial M_i / \partial N_j < 0$,

(iii) $\partial M_i / \partial \alpha_i > 0$, $\partial M_i / \partial \alpha_j < 0$,

(iv) $\partial^2 M_i / \partial N_i \partial \alpha_i > 0$ and $\partial^2 M_i / \partial N_i \partial \alpha_j < 0$, and

(v) the sign of $\partial^2 M_i / \partial N_i \partial N_j$ is ambiguous.

Proof. First, note that we can write $M_i = (\beta - \gamma + \gamma N_i)\Lambda_i/Z_i$ since $X_i = \Lambda_i/Z_i$. Thus, using (B.2)-(B.4) leads to

$$\frac{\partial M_i}{\partial N_i} = \frac{\gamma \Lambda_i (\beta - \gamma)\Phi_{-i}}{Z_i^2} > 0. \tag{B.24}$$

Moreover, since $\partial Z_i / \partial N_i > 0$, according to (B.2), (B.24) implies $\partial^2 M_i / \partial N_i^2 < 0$. This confirms part (i) of Corollary B.2. In a similar fashion as in the proof of part (ii) of Corollary B.1, one can also show that

$$\frac{\partial M_i}{\partial N_j} = -\frac{2\gamma(\beta - \gamma)(\beta - \gamma + \gamma N_i)S_{i,j}}{[2(\beta - \gamma) + \gamma N_j]^2 Z_i^2} < 0, \tag{B.25}$$

$j \neq i$. Part (iii) follows directly from recalling $\partial \Lambda_i / \alpha_i > 0$ and $\partial \Lambda_i / \alpha_j < 0$, $j \neq i$, together with $\partial M_i / \partial \alpha_j = (\beta - \gamma + \gamma N_i)\partial X_i / \partial \alpha_j$ and $X_i = \Lambda_i/Z_i$. Part (iv) follows from (B.24), and, again, $\partial \Lambda_i / \alpha_i > 0$ and $\partial \Lambda_i / \alpha_j < 0$, $j \neq i$. Finally, using (B.24), together with (B.7)-(B.9), one can show that, for $j \neq i$,

$$\frac{\partial^2 M_i}{\partial N_i \partial N_j} = \frac{2\gamma^2(\beta - \gamma)^2 \{(\alpha_i - \alpha_j)\Phi_{-i}Z_i + \Lambda_i [2(\beta - \gamma + \gamma N_i) - \Phi_{-i}(2(\beta - \gamma) + \gamma N_i)]\}}{[2(\beta - \gamma) + \gamma N_j]^2 Z_i^3}. \tag{B.26}$$

Unfortunately, for $j \neq i$, the sign of $\partial^2 M_i / \partial N_i \partial N_j$ is ambiguous even for $\alpha_i = \alpha_j$. This concludes the proof of Corollary B.2. ∎

2 Bertrand case under linear demand

Proposition C.1 (Equilibrium at stage 2 in Bertrand competition under (1)). *In an interior Bertrand-Nash equilibrium at stage 2, profits are given by*

$$\Pi_i = \frac{N_i \left(\beta - \gamma + \gamma \sum_{j \neq i} N_j\right)}{\left[2(\beta - \gamma) + \gamma \sum_{j \neq i} N_j\right]^2} \left(\frac{\Theta_i}{1 + \sum_i \Omega_i}\right)^2, \tag{C.1}$$

where

$$\Omega_i \equiv \frac{\gamma N_i \left(\beta - \gamma + \gamma \sum_{j \neq i} N_j\right)}{(\beta - \gamma) \left(2(\beta - \gamma) + \gamma \sum_{j \neq i} N_j\right)} \tag{C.2}$$

and $\Theta_i \equiv \alpha_i \left(1 + \sum_{j \neq i} \Omega_j\right) - \sum_{j \neq i} \alpha_j \Omega_j$.

Proof. First, note that $\pi_i = \sum_{k \in \mathcal{N}_i} (p_k - c_i) x_k$ implies

$$\frac{\partial \pi_i}{\partial p_k} = x_k + \sum_{l \in \mathcal{N}_i} (p_l - c_i) \frac{\partial x_l}{\partial p_k}, \tag{C.3}$$

where

$$\frac{\partial x_l}{\partial p_l} = -\frac{\beta - \gamma + \gamma(K - 1)}{(\beta - \gamma)\beta - \gamma + \gamma K} \tag{C.4}$$

and, for $l \neq k$,

$$\frac{\partial x_l}{\partial p_k} = \frac{\gamma}{(\beta - \gamma)\beta - \gamma + \gamma K} \tag{C.5}$$

according to demand structure (1). Thus, optimal behavior of firm $i \in \mathcal{I}$ at stage 2 is given by the following set of first-order conditions (presuming an interior solution):

$$
\begin{aligned}
0 = \ & x_k - [\alpha_i - (\beta - \gamma)x_k - \gamma Q] \frac{\beta - \gamma + \gamma(K - 1)}{(\beta - \gamma)\beta - \gamma + \gamma K} + \\
& \frac{\gamma}{(\beta - \gamma)\beta - \gamma + \gamma K} \sum_{l \in \mathcal{N}_i \setminus \{k\}} [\alpha_i - (\beta - \gamma)x_l - \gamma Q],
\end{aligned} \tag{C.6}
$$

$k \in \mathcal{N}_i$, where again $Q = \sum_{l \in \mathcal{K}} x_l$. Imposing $x_k = X_i$ for all $k \in \mathcal{N}_i$, it is straighforward to show that (C.6) implies

$$X_i = \frac{\beta - \gamma + \gamma(K - N_i)}{2(\beta - \gamma) + \gamma(K - N_i)} \frac{\alpha_i - \gamma Q}{\beta - \gamma}. \tag{C.7}$$

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Thus, as $K - N_i = \sum_{j \neq i} N_j$, using the definition of Ω_i in Proposition C.1, we have $\gamma N_i X_i = (\alpha_i - \gamma Q) \Omega_i$. Summing over all $i \in \mathcal{I}$ and using $Q = \sum_i N_i X_i$, one obtains

$$\gamma Q = \frac{\sum_i \alpha_i \Omega_i}{1 + \sum_i \Omega_i}, \tag{C.8}$$

and thus

$$\alpha_i - \gamma Q = \frac{\Theta_i}{1 + \sum_i \Omega_i}, \tag{C.9}$$

where Θ_i is defined in Proposition C.1. Combining (C.7) and (C.9) yields

$$X_i = \frac{\beta - \gamma + \gamma \sum_{j \neq i} N_j}{2(\beta - \gamma) + \gamma \sum_{j \neq i} N_j} \frac{\Theta_i}{(\beta - \gamma)(1 + \sum_i \Omega_i)}. \tag{C.10}$$

Now substitute both (C.9) and (C.10) into $p_k - c_i = \alpha_i - \gamma Q - (\beta - \gamma)X_i [= M_i]$, which holds for all $k \in \mathcal{N}_i$ (compare with the proof of Proposition 1). This yields

$$M_i = \frac{\beta - \gamma}{2(\beta - \gamma) + \gamma \sum_{j \neq i} N_j} \frac{\Theta_i}{1 + \sum_i \Omega_i} \tag{C.11}$$

Finally, noting that $\pi_i = N_i X_i M_i$ and using (C.10) and (C.11) confirms Proposition C.1. ∎

Lemma C.1. *In an interior Bertrand-Nash equilibrium at stage 2, if α_i is relatively low, then mark-up M_i is decreasing in N_i. For instance, in a duopoly, $\partial M_1 / \partial N_1 < 0$ if $\alpha_1 \leq \alpha_2$.*

Proof. Acording to (C.11), we have

$$\frac{\partial M_i}{\partial N_i} = \frac{\beta - \gamma}{2(\beta - \gamma) + \gamma \sum_{j \neq i} N_j} \frac{\frac{\partial \Theta_i}{\partial N_i}(1 + \sum_i \Omega_i) - \Theta_i \left(\frac{\partial \Omega_i}{\partial N_i} + \sum_{j \neq i} \frac{\partial \Omega_j}{\partial N_i} \right)}{(1 + \sum_i \Omega_i)^2}. \tag{C.12}$$

According to (C.1), $\partial \Omega_i / \partial N_i = \Omega_i / N_i > 0$ and, for $j \neq i$,

$$\frac{\partial \Omega_j}{\partial N_i} = \frac{\gamma^2 N_j}{\left(2(\beta - \gamma) + \gamma \sum_{h \neq j} N_h \right)^2} > 0. \tag{C.13}$$

Moreover, from the definition of Θ_i, we have $\partial \Theta_i / \partial N_i = \alpha_i \sum_{j \neq i} \partial \Omega_j / \partial N_i - \sum_{j \neq i} \alpha_j \partial \Omega_j / \partial N_i$. Hence, if α_i is relatively low, or in case of dupoly if $\alpha_i \leq \alpha_j$, then $\partial \Theta_i / \partial N_i \leq 0$ and thus $\partial M_i / \partial N_i < 0$. This concludes the proof. ∎

Derivation of $\partial^2\Pi_1/\partial N_1\partial\alpha_1$ **and** $\partial^2\Pi_1/\partial N_1\partial\alpha_2$**:** Proposition C.1 implies for the duopoly case that

$$\Pi_1 = \frac{(\beta - \gamma + \gamma N_2)\,N_1}{(2(\beta - \gamma) + \gamma N_2)^2}\frac{\alpha_1 + (\alpha_1 - \alpha_2)\Omega_2}{1 + \Omega_1 + \Omega_2} \tag{C.14}$$

where

$$\Omega_1 = \frac{\gamma N_1(\beta - \gamma + \gamma N_2)}{(\beta - \gamma)(2(\beta - \gamma) + \gamma N_2)} \text{ and } \Omega_2 = \frac{\gamma N_2(\beta - \gamma + \gamma N_1)}{(\beta - \gamma)(2(\beta - \gamma) + \gamma N_1)}. \tag{C.15}$$

From this, it is straightforward to show that

$$\begin{aligned}
\frac{\partial\Pi_1}{\partial N_1} &= \frac{(\beta - \gamma + \gamma N_2)\left[\alpha_1 + (\alpha_1 - \alpha_2)\Omega_2\right]}{(2(\beta - \gamma) + \gamma N_2)^2} \times \\
&\quad \{(\alpha_1 - \alpha_2)[2N_1(1 + \Omega_1)\partial\Omega_2/\partial N_1 + \Omega_2(1 + \Omega_2 - \Omega_1)] + \\
&\quad \alpha_1[1 + \Omega_2 - \Omega_1 - 2N_1\partial\Omega_2/\partial N_1]\}
\end{aligned} \tag{C.16}$$

Recall that $\partial\Omega_2/\partial N_1 > 0$, according to (C.13). Thus, if $\alpha_1 = \alpha_2$, then $1 + \Omega_2 - \Omega_1 > 0$ is a necessary condition for $\partial\Pi_1/\partial N_1 > 0$ to hold. One can now rewrite (C.16) as

$$\begin{aligned}
\frac{\partial\Pi_1}{\partial N_1} &= \frac{(\beta - \gamma + \gamma N_2)\left[\alpha_1 + (\alpha_1 - \alpha_2)\Omega_2\right]}{(2(\beta - \gamma) + \gamma N_2)^2} \times \\
&\quad \{\alpha_1\left[(1 + \Omega_2)(1 + \Omega_2 - \Omega_1) + 2N_1\Omega_1\partial\Omega_2/\partial N_1\right] - \\
&\quad \alpha_2[2N_1(1 + \Omega_1)\partial\Omega_2/\partial N_1 + \Omega_2(1 + \Omega_2 - \Omega_1)]\}.
\end{aligned} \tag{C.17}$$

Hence, $\partial^2\Pi_1/\partial N_1\partial\alpha_1 > 0$ and $\partial^2\Pi_1/\partial N_1\partial\alpha_2 < 0$ if $1 + \Omega_2 - \Omega_1 > 0$. This confirms the claim in section 4.2 that, in the neighborhood of a symmetric equilibrium, we have $\partial^2\Pi_1/\partial N_1\partial\alpha_1 > 0$ and $\partial^2\Pi_1/\partial N_1\partial\alpha_2 < 0$. ∎
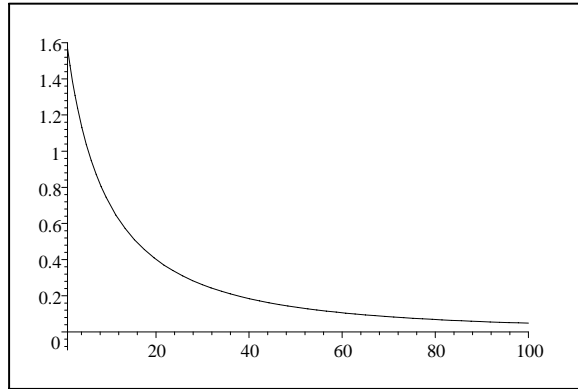
Numerical Analysis: Specifying $\alpha_1 = \alpha_2 = 10$, it is easy to show from (C.14) and (C.15) that

$$\frac{\partial\Pi_1}{\partial N_1} = \frac{100\,(\beta - \gamma + \gamma N_2)\left(1 + \frac{\gamma N_1(\beta - \gamma + \gamma N_2)}{(\beta - \gamma)(2(\beta - \gamma) + \gamma N_2)} - \frac{\gamma N_2(\beta - \gamma + \gamma N_1)}{(\beta - \gamma)(2(\beta - \gamma) + \gamma N_1)} - \frac{2\gamma^2 N_1 N_2}{(2(\beta - \gamma) + \gamma N_1)^2}\right)}{(2(\beta - \gamma) + \gamma N_2)^2\left(1 + \frac{\gamma N_1(\beta - \gamma + \gamma N_2)}{(\beta - \gamma)(2(\beta - \gamma) + \gamma N_2)} + \frac{\gamma N_2(\beta - \gamma + \gamma N_1)}{(\beta - \gamma)(2(\beta - \gamma) + \gamma N_1)}\right)^3}. \tag{C.18}$$
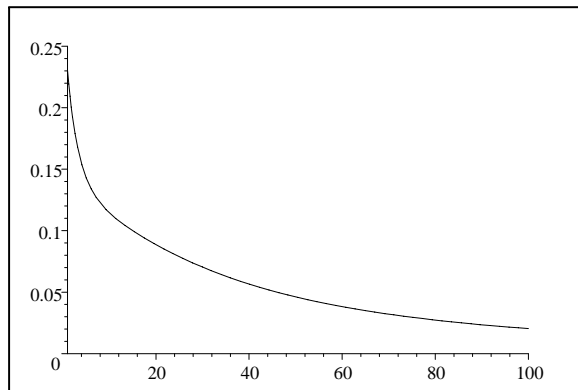
The following graphs illustrate $\partial\Pi_1/\partial N_1$ as function of N_1 and N_2, respectively. We start with specifications $\beta = 10$ and $\gamma = 1$ and examine whether $\partial\Pi_1/\partial N_1 > 0$ and $\partial^2\Pi_1/\partial N_1^2 < 0$ for various product ranges of the rival firm, N_2.
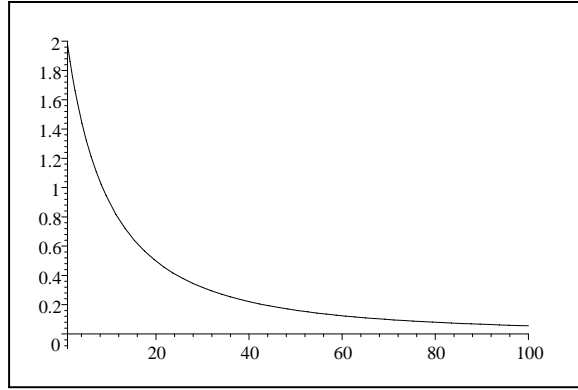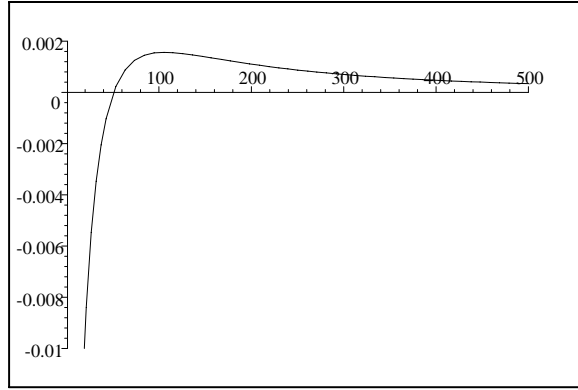
$\frac{\partial \Pi_1}{\partial N_1}$ for $N_2 = 1$, $\beta = 10$, $\gamma = 1$

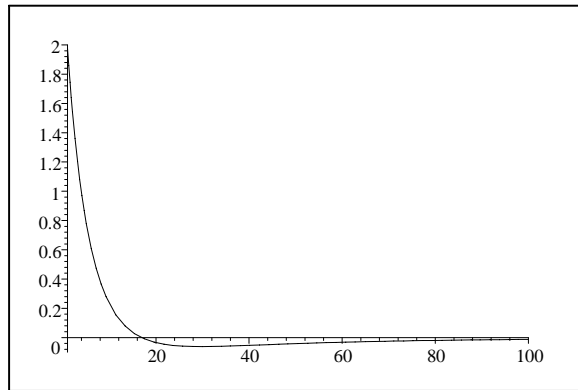$\frac{\partial \Pi_1}{\partial N_1}$ for $N_2 = 2$, $\beta = 10$, $\gamma = 1$

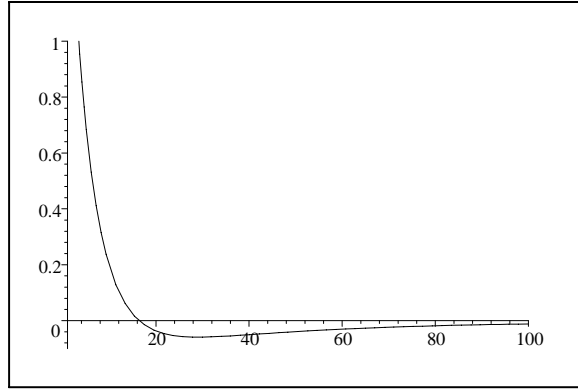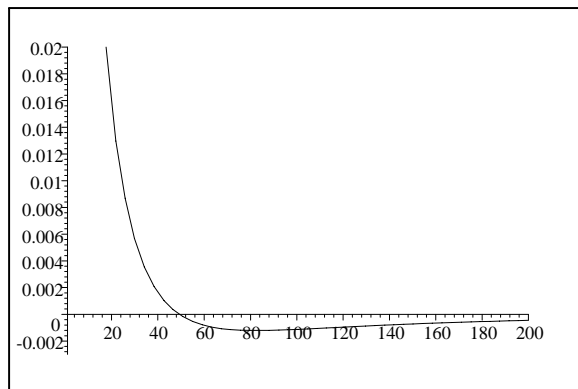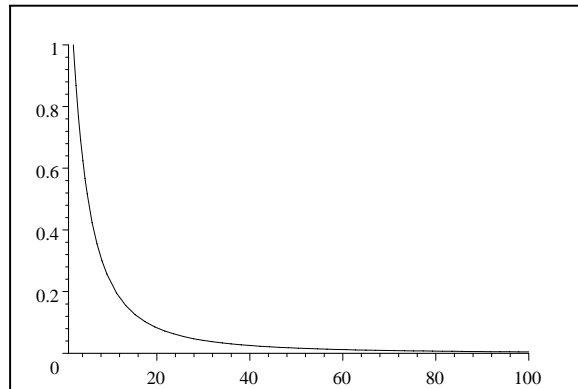$\frac{\partial \Pi_1}{\partial N_1}$ for $N_2 = 10$, $\beta = 10$, $\gamma = 1$

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$\frac{\partial \Pi_1}{\partial N_1}$ for $N_2 = 50$, $\beta = 10$, $\gamma = 1$

The preceding graphs suggest that $\partial \Pi_1 / \partial N_1 > 0$ and $\partial^2 \Pi_1 / \partial N_1^2 < 0$ if N_2 is sufficiently small or if N_1 is high enough. For instance, $\partial \Pi_1 / \partial N_1 > 0$ and $\partial^2 \Pi_1 / \partial N_1^2 < 0$ hold for all N_1 if $N_2 \leq 10$, but for $N_2 = 50$ only if N_1 is high.

Next, consider $\partial \Pi_1 / \partial N_1$ as function of N_2, holding N_1 constant.

$\frac{\partial \Pi_1}{\partial N_1}$ for $N_1 = 1$, $\beta = 10$, $\gamma = 1$

$\frac{\partial \Pi_1}{\partial N_1}$ for $N_1 = 2$, $\beta = 10$, $\gamma = 1$
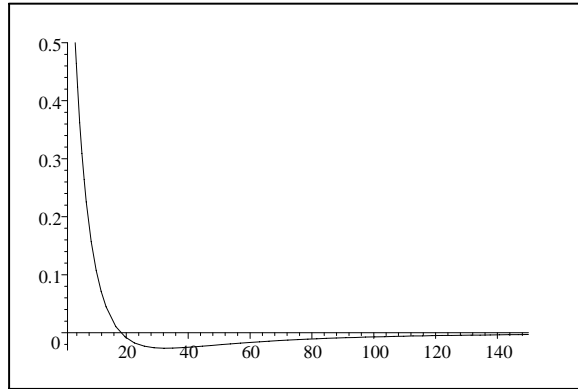
$\frac{\partial \Pi_1}{\partial N_1}$ for $N_1 = 10$, $\beta = 10$, $\gamma = 1$

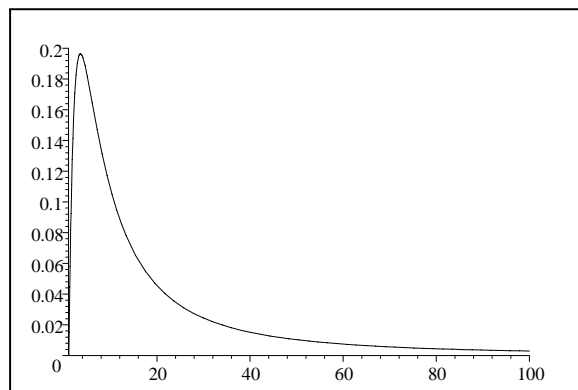

$\frac{\partial \Pi_1}{\partial N_1}$ for $N_1 = 50$, $\beta = 10$, $\gamma = 1$

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One thus finds that $\partial^2\Pi_1/\partial N_1\partial N_2 < 0$ whenever $\partial\Pi_1/\partial N_1 > 0$. Hence, the same conditions which lead to an incentive to launch new varieties also ensure negatively sloped reaction functions at stage 1, confirming the claims in the main text (section 4.2).
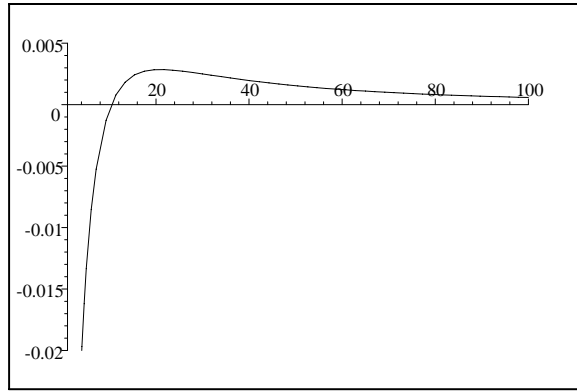
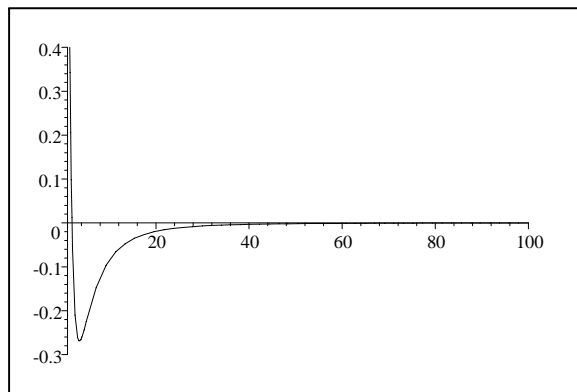Now let us consider $\beta = 2$ and still keep $\gamma = 1$.

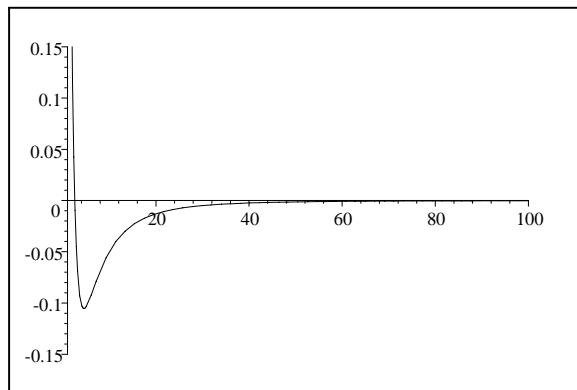$\dfrac{\partial\Pi_1(N_1,\cdot)}{\partial N_1}$ for $N_2 = 1$, $\beta = 2$, $\gamma = 1$

$\dfrac{\partial\Pi_1}{\partial N_1}$ for $N_2 = 2$, $\beta = 2$, $\gamma = 1$
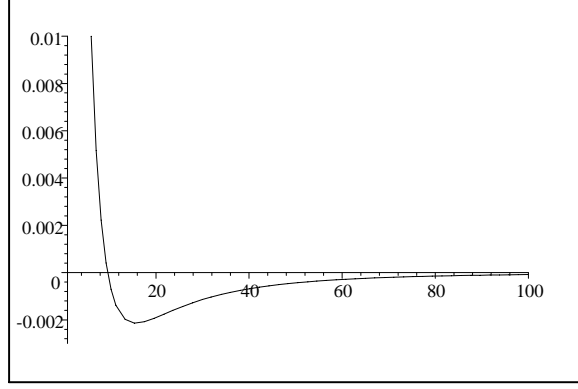
$\frac{\partial \Pi_1}{\partial N_1}$ for $N_2 = 10$, $\beta = 2$, $\gamma = 1$

$\frac{\partial \Pi_1}{\partial N_1}$ for $N_1 = 1$, $\beta = 2$, $\gamma = 1$

$\frac{\partial \Pi_1}{\partial N_1}$ for $N_1 = 2$, $\beta = 2$, $\gamma = 1$

$\frac{\partial \Pi_1}{\partial N_1}$ for $N_1 = 10$, $\beta = 2$, $\gamma = 1$

Unsurprisingly, the incentive to launch varieties is lower if $\beta = 2$ than if $\beta = 10$, as products are better substitutable in the former case. But the conclusions from the case $\beta = 10$ drawn above remain valid.

Proof that if $\alpha_1 > \alpha_2$ implies $N_1^* > N_2^*$, then it also implies $D_1^* > D_2^*$: First, recall that $D_i = N_i X_i$ when all varieties within a firm's product line are produced in same quantity. Thus, using (C.8),

$$
\begin{aligned}
D_1 &= \frac{N_1 \left(\beta - \gamma + \gamma N_2\right)}{2(\beta - \gamma) + \gamma N_2} \frac{\Theta_1}{(\beta - \gamma)\left(1 + \Omega_1 + \Omega_2\right)}, \\
D_2 &= \frac{N_2 \left(\beta - \gamma + \gamma N_1\right)}{2(\beta - \gamma) + \gamma N_1} \frac{\Theta_2}{(\beta - \gamma)\left(1 + \Omega_1 + \Omega_2\right)}.
\end{aligned}
\tag{C.19}
$$

Hence, we have $D_1 > D_2$ if, for instance, $\Theta_1 > \Theta_2$ and

$$
\frac{\beta - \gamma + \gamma N_2}{2(\beta - \gamma) + \gamma N_2} \geq \frac{\beta - \gamma + \gamma N_1}{2(\beta - \gamma) + \gamma N_1}.
\tag{C.20}
$$

It is easy to confirm that (C.20) holds if and only if $N_1 \geq N_2$. Moreoover, analogously to (A.16) derived in the proof of Proposition 4, one can show from the definition of Θ_i that

$$
\Theta_1 - \Theta_2 = \left(\alpha_1 - \alpha_2\right)\left(1 + \Omega_1 + \Omega_2\right),
\tag{C.21}
$$

i.e., $\Theta_1 > \Theta_2$ if $\alpha_1 > \alpha_2$. Thus, if $\alpha_1 > \alpha_2$ implies $N_1^* > N_2^*$, then it also implies $D_1^* > D_2^*$, as claimed in section 4.2. ∎

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